Research Paper

Plane Stress Analysis of Layered Composite Plate subjected to varying in-plane patch load

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Abstract

Composites are very successful in resisting the depletion brought about by excessive dynamic conditions. Despite the fact that the segments made out of composites are lean, in view of its sti ness and high strength properties they are widely utilized. This paper aims at the comparison of each in-plane stress against corresponding allowable stresses within a lamina which makes use of maximum stress criterion method. In a lamina, when the stress value at a material point exceeds ber strength the failure along the ber direction is declared. Similarly, when the corresponding stress value exceeds matrix normal strength the failure in transverse direction to ber within a lamina is con rmed. Applying the nite element analysis the rst ply failure in multi-layered composites is determined. As a part of this work an in-house code was developed using Mat Lab script. An adaptive mesh was considered to account for in-plane stress concentrations in the laminate. A mesh sensitivity study was also conducted to ensure convergence of solution for the problem domain. Validation of the formulation and implementation was conducted by utilizing di erent loading conditions. In particular, a transverse load was applied and the de ection was compared against closed form solution to ensure correctness.

Keywords: nite element, plane stress, de ection, isotropic, along ber direction, across ber direction.

1 Introduction

In the last few decades composite materials have become a paramount engineering material in various elds. Composite materials have a lower shear modulus in comparison with the extensional rigidity which makes the material weak in shear. Hence, the role of transverse shear is a requisite in composites. Therefore, precise perception of structural behaviour of composites such as displacements and stresses is required. In the present work, the aim of study is to inspect maximum stress in the composite plates under varying loading. To calculate the laminate-stresses in a laminated composite plate a model in nite element with ve degrees of freedom is developed and studied.

The present study aims at understanding about the failure mechanisms and to draw an algorithm for the process of analysis and also to write the required code in MATLAB.

The main objectives of this study are the following:
a) To perform plane stress analysis of composite laminate subjected to a wide variety of loads;
b) To identify the rst failure location of position and the layer number;
c) To perform a parametric study of width of edge load on stress distribution and to present graphically in the form of contours.

2 Methodology

Initially the sti ness matrix in principal material direction for each lamina is worked out which has been transformed to the global direction. Sti ness matrix is initiated for the whole laminate by using Mindlin theory. By selecting the proper shape functions eight nodded isoparametric element is applied to develop strain displacement matrix for an element. Element sti ness matrix is generated by making use of displacement [B] and strain displacement matrix [C],

$$[k_p]_e = [B]^T_j [C]_j [B]_j \, dA \qquad (1)$$

where k_p is sti ness matrix and dA is area of the element. Strain energy $U_l p$ due to linear strain is given by

$$U_l p = \frac{1}{2} \sum_{j=1} \{d\}^T_j [k_p]_e \{d\}_j \qquad (2)$$

in which $\{d\}_j$ is displacement vector for j^{th} element.

The above summations are made in the sense of nite element assemblage, taking the global displacement vector to be d, which results in the following relation.

$$U_l p = \frac{1}{2} \{d\}^T [k_p]_e \{d\} \qquad (3)$$

in which $[k_p]_e$ is the structure or global elastic sti ness matrix for the plate. If Π is potential energy then using the principle of minimum potential energy, variation of Π should be minimum.

or

$\delta\Pi = 0;$

or

$$\delta\Pi = \{\delta d\}^T [k_p] \{d\} - \{\delta d\}^T \{F\} = 0 \qquad (4)$$

or

$$\delta\Pi = \{\delta d\}^T \left([k_p] \{d\} - \{F\} \right) = 0 \qquad (5)$$

since δd is arbitrary, it cannot be zero.

Hence,

$$[k_p]\{d\} - \{F\} = 0 \quad (6)$$

or

$$[k_p]\{d\} = \{F\} \quad (7)$$

where F is the lateral load vector, and d is the displacement vector. This is the governing equation for the plane stress analysis.

Strain at every Jacobian point is generated by using displacement vector. Extrapolation is done for mid-plane strains at jacobian points to the four corner nodes of corresponding element. Generation of stresses at all the nodes by using strain at each layer and corresponding modi ed reduced sti ness matrix, $\sigma = Qs$ [1, 2].
The average stresses at every node in each layer are found by making use of top and bottom stresses of four corners. Stresses are transformed from global directions to stresses along and across the ber direction. Stress distribution is studied by plotting the stress contours.

3 Results and Discussion

The 8-nodded isoparametric element with each node of ve degrees of freedom is considered. The plate is divided into 10x10 elements and each element has 8 nodes as in gure 1, which shows node numbering in mesh of 10x10 for a composite plate of size 100x100.
The discussion on classical and numerical formulation of the problem pertaining to the present study has been done. After ensuring the validation of coding and formulation, detailed studies have been carried out for lamina failure and stress contours are developed for a plate with 10 x 10 elements.

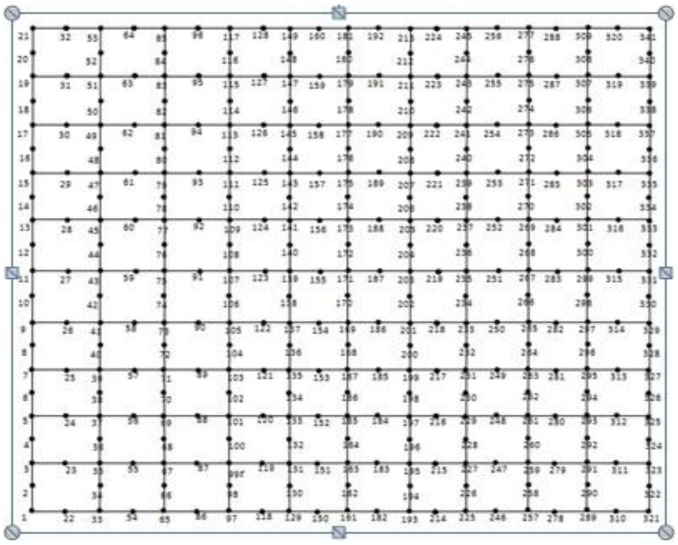

Figure 1: Node numbering in mesh of 10x10 for a composite plate of size 100x100.

Figure 2 shows geometrical con guration of laminated plate.
alx - Dimension of plate along x direction
bly - Dimension of plate along y direction
atz - Thickness of Plate
D - Flexural rigidity, $D = \frac{E(atz)^3}{12(1-\vartheta^2)}$ for isotropic plate [3]
and $D = \frac{E_{22}(atz)^3}{12(1-\vartheta^2_{12})}$ for composite plate.
nx - number of divisions in x-direction
my - number of divisions in y-direction
E_{11} - Young's modulus in the ber direction
E_{22} - Young's modulus in the lateral direction of the ber
tt_{12} - in-plane shear modulus
ν_{12} - major Poisson's ratio
θ - orientation of ber in a layer
α - plate aspect ratio, (alx)/(bly)
β - plate width to thickness ratio, bly/atz

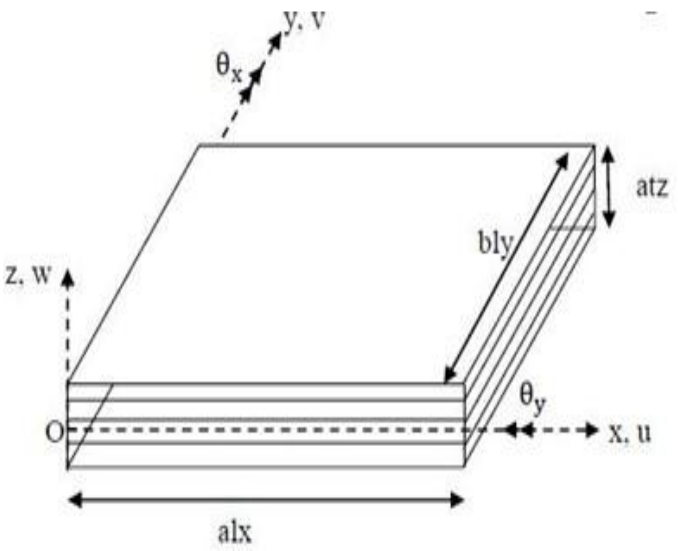

Figure 2: Geometrical con guration of laminated plate.

Table 1: Material properties.

2*Material	Material Constants		
	υ_{12}	G_{23}	G_{13}
Composite (M-1)	0.25	$0.5E_2$	$0.6E_2$
Isotropic (M-2)	0.30	0.3846	0.3846
2*	Material Constants		
	G_{12}	E_2	E_1
Compositer (M-1)	$0.6E_2$	1.0	40.0
Isotropic (M-2)	0.3846	1.0	1.0

The load application on a cantilever plate of size 100x100 is shown in gure 3. Table 1 gives the material properties of isotropic as well as composite plates.

3.1 Validation of elastic sti ness matrix

The element sti ness matrix is of size 40 x 40 and the manual calculation becomes quite complicated. By considering a plate displacement acted upon by lateral load structure sti ness matrix validation is done. The relation picked up is the bending problem relation which is given in equation 7. The plate considered for validation is a square plate with all sides simply supported. It is subjected to a lateral UDL of magnitude q/unit area. The plate gets de ected and maximum de ection ($W_{\max}$) [4] will occur at centre and it is made as non-dimensional and is given as,

$$\delta = \frac{W_{\max} x D}{q x a^4} \tag{8}$$

where δ = non dimensional de ection, D= exural rigidity, q=lateral load applied on plate, and a =length of plate.

Table 2: Convergence study of central de ection of isotropic square plate subjected to lateral uniform distributed load simply supported in all sides.

Central de ection exact value (δ[10])	Mesh size (m × n)	Max. central de ection FEM value (δ)
5*0.00406	2 × 2	0.00189
	4 × 4	0.00401
	6 × 6	0.00405
	8 × 8	0.00406
	10 × 10	0.00406

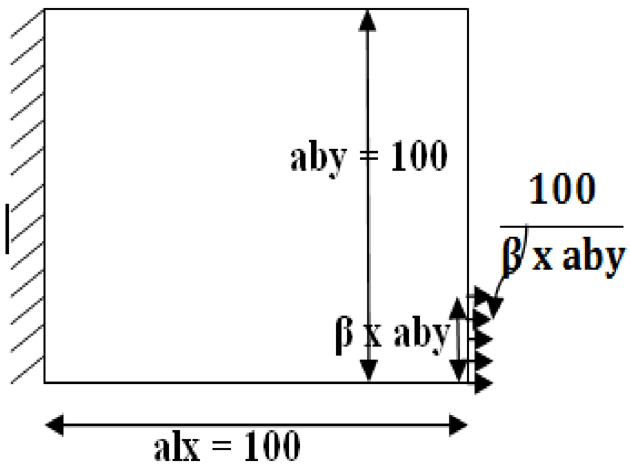

Figure 3: Load application on a cantilever plate of size 100x100.

Table 2 shows convergence study of central de ection of isotropic square plate subjected to lateral uniform distributed load simply supported in all sides. Accurate result is obtained for 10 x 10 meshes and hence, this mesh is used for all analysis. The size of composite plate considered is 100 x 100 and is of unit thickness. The plate is xed at left side and all other sides are free. A tensile patch load of 100 units is applied at right end in x direction. Patch load is applied from one end of the plate and the width is increased in steps. The contour maps for various patch loads of 0.2, 0.4, 0.6 and 0.8 are obtained only for 2 layered anti-symmetric laminate.

Shown in gures 4-11 are stress contours for σ_1 and σ_2, plotted for 2 layered anti-symmetric composite plate for layers 1 and 2 by varying patch loads from 0.2 to 0.8 with an increase of 0.2, where σ_1 = stress along bre direction and σ_2 = stress across bre direction.

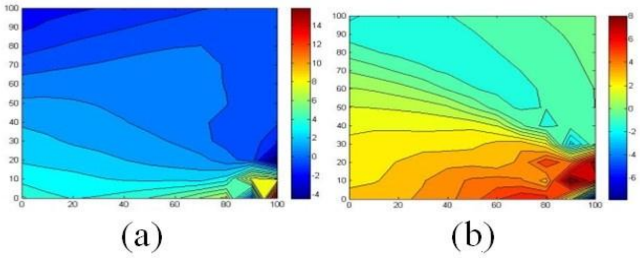

Figure 4: σ_1 distribution for patch load=0.2 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

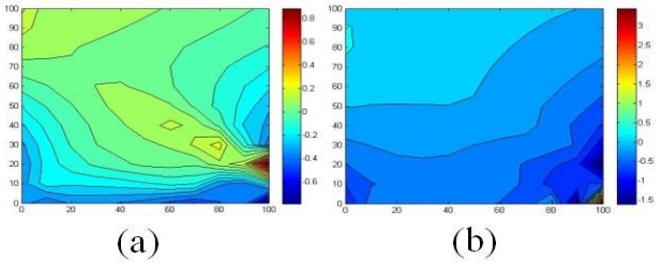

Figure 5: σ_2 distribution for patch load=0.2 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

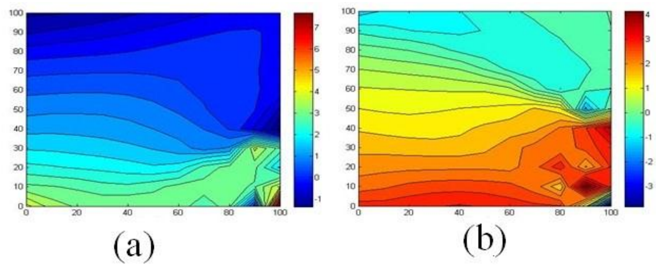

Figure 6: σ_1 distribution for patch load=0.4 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

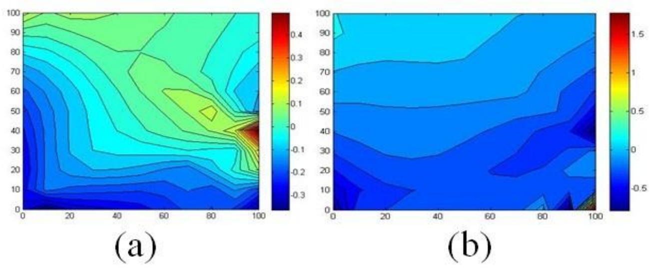

Figure 7: σ_2 distribution for patch load=0.4 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

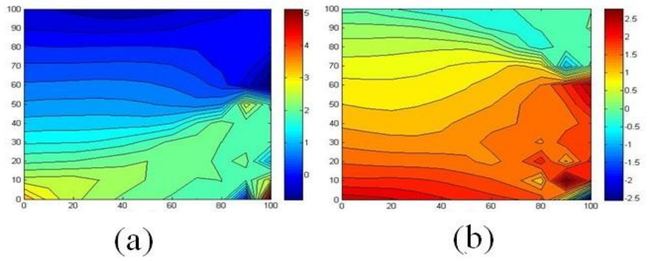

Figure 8: σ_1 distribution for patch load=0.6 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

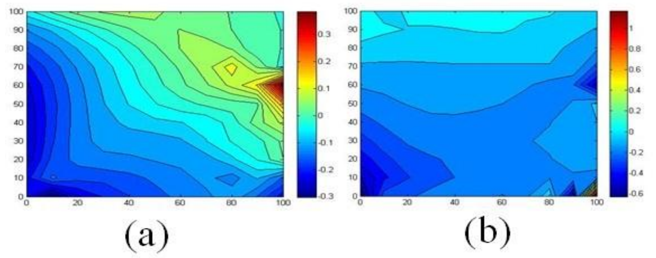

Figure 9: σ_2 distribution for patch load=0.6 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

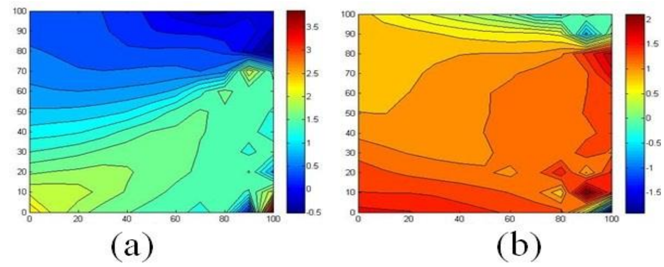

Figure 10: σ_1 distribution for patch load=0.8 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

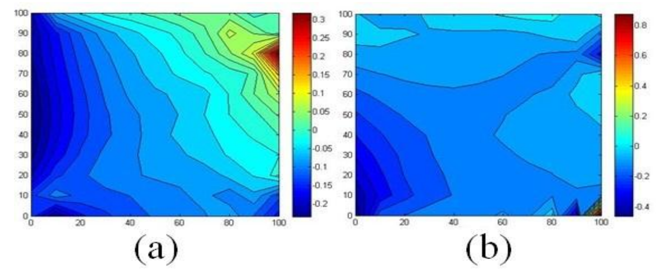

Figure 11: σ_2 distribution for patch load=0.8 for a two layered (30/-30)anti-symmetric composite plate: (a) in layer 1 and (b) in layer 2.

For patch load=0.2, both σ_1 and σ_2 show higher values near the load region. In layer 1, σ_1 changes from tensile to compressive as the distance from the load region increases and the region at top left corner is subjected to high compressive stress. Along ber direction, tensile stress developed is less in layer 1 as compared to layer 2 whereas maximum compressive stress is in layer 2.

For patch load=0.4, layer 1 and 2 are subjected to compressive stress at the top and tensile stress at bottom in ber direction. Both σ_1 and σ_2 show higher values near the load region.
In layer 1, σ_1 changes from tensile to compressive as the distance from the load region increases and the region at top left corner is subjected to high compressive stress. Along ber direction, tensile stress developed is less in layer 1 compared to layer 2 whereas maximum compressive stress is found in layer 2.

For patch load=0.6, major portion of layers are subjected to tensile ber direction stress as the tensile load is applied for 0.6 times the width from bottom at right end. Stresses developed in layer 1 are more in comparison with layer 2. Stresses across the ber direction are compressive for major portion of the layer.

For patch load=0.8, major portion of layers are subjected to tensile ber direction stress as the tensile load is applied for 0.8 times the width from bottom at right end. Stresses developed in layer 1 are more in comparison with layer 2. Stresses across the ber direction are compressive for major portion of the layer. It is tensile near the end point of location of load.

4 Conclusion

The load de ection phenomena obtained from the coding has been validated with the results obtained from the classical solution. For all cases considered in this work, general ber direction stresses are tensile near the loading region and compressive in top region. Maximum stresses have been found near the end of the load location. In general, across ber direction stresses are compressive near loading region and tensile in the region away from load. Stresses across the ber direction are less in comparison with the stresses along ber direction. In almost all cases maximum stresses are developed in the corner element near the loading region.

References

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