Scientific Article

Search for the value of π A brief history

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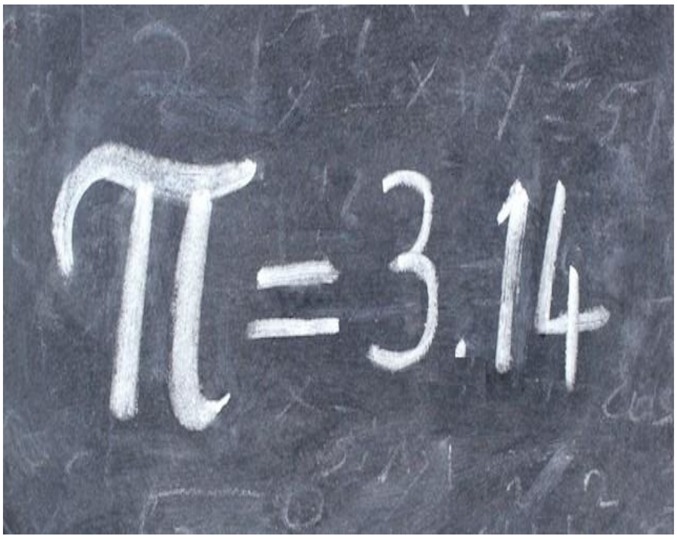
The calculation of the ratio between perimeter of a circle and its diameter, which is known by the Greek letter π, has been one of the most enduring challenges in mathematics.



From ancient Babylonia, Egypt, China, India, Middle Ages in Europe and to the present day supercomputers, mathematicians have been trying to calculate the value of π which has evoked much mystery, romanticism and interest. Although we may never know the origin of π, the people who initiated the hunt for the value of π were the Babylonians and Egyptians, who measured experimentally the ratio of the perimeter and diameter of a circle with a string nearly 4000 years ago. Antiphon and Bryson, in ancient Greece, took up the problem with an innovative idea of tting a polygon inside a circle, calculating its area, and doubling the sides repeatedly. However, their work resulted in nding the areas of many tiny triangles, which was not only complicated but also resulted in few digits [1]. The rst Greek whose work initiated a major impact in the calculation of π was Archimedes of Syracuse. Unlike Antiphon and Bryson, Archimedes took up the challenge by focusing on the perimeter of polygons as opposed to their areas and approximated the circumference of the circle instead of its area. He inscribed a hexagon in a circle and doubled its sides four times to nish with a 96-sided polygon; after numerous calculations he ended with a result:
$3\,10/71 < \pi < 3\,1/7$ which has lower and upper bounds for the value of π [2]. While the earliest value of π considered in China was 3, Liu Hui independently calculated, in 263 AD, the perimeters of regular inscribed polygons with upto 192 sides, and derived the value = 3.14159, which is accurate to the rst ve digits; this is similar to the method used by Bryson and Antiphon hundreds of years earlier in Greece. Subsequently, Tsu Chung-chih and his son Tsu Keng-chih, at the end of the 5^{th} century calculated the value of π by using inscribed polygons with 24,576 sides and came up with upper and lower bounds as $3.1415926 < \pi < 3.1415927$! [1] Later, Aryabhata from India gave the 'precise' value 62,832/20,000 = 3.1416 [3]. Soon after, Brahmagupta, another Indian mathematician, used a novel approach by calculating the perimeters of polygons inscribed in a circle and postulated that the value of π would approach the square root of 10 [=3.162...]. In Europe in 1593, Adrianus Romanus used an inscribed polygon with 230 sides to calculate value of π upto 17 digits. Later, Ludolph Van Ceule from Germany used the Archimedes's method of polygons with over 500 million sides and came up with 20 digits; moving on, in 1610, he calculated value of π accurate to 35 digits. His accomplishments, realized without the use of a modern computer, were considered so important that the digits were engraved on his tombstone in St. Peter's Churchyard in Leyden [4].

In twentieth century, D. F. Ferguson discovered an earlier error (1873) in William Shanks calculation of value of π from the 528^{th} digit onward. In 1947, he presented the results of his calculations with 808 digits of π [5]. Later, Levi Smith and John Wrench achieved the 1000-digit-mark [5]. A new breakthrough which emerged in 1949 is with the invention of ENIAC (Electronic Numerical Integrator and Computer); using punch cards and ENIAC, a group of mathematicians calculated 2037 digits in just seventy hours [3]. The advent of the electronic computer in the 1970s resulted in a race to nd the accurate value of π [1]. A breakthrough occurred in 1976 when Eugene Salamin found an algorithm which doubles the number of precise digits with each iteration; this is an improvement over the previous formulae which only added a few digits per calculation [1]. Over the next twenty years many researchers worked together and led the way throughout the 1980s, until in August 1989 David and Gregory Chudnovsky broke the one-billion-digit-barrier. Kanada and Takahashi in 1997, calculated 51.5 billion digits in just over 29 hours, which means an average rate of about 500,000 digits per second!. This was further improved by them in 1999 to 68,719,470,000 digits. [1]. The question is whether calculation of π to the n^{th} digit (where n > one billion) is really necessary?? Just 39 decimal places would be su cient to calculate the perimeter of a circle surrounding the known universe with a precision of the radius of a hydrogen atom!! At the present time, it appears that the only tangible application of all those digits is to nd the bugs in computer chips and computers. Further, mathematicians are also looking for the occurrence of some rules that will distinguish the digits of π from other numbers. But, Chudnovskys have also opined that no other calculated number comes close to a random sequence of digits other than that of magical number π.

References

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